1. (e) In each of the situations in answers a-d, the person and the frame of reference is subject to an acceleration. In an accelerated reference frame Newton’s law of inertia is not valid, so the reference frame is not an inertial reference frame.

2. (a) The worker measures the proper time, because he is at rest with respect to the light and views the flashes as occurring at the same place.

3. 1.89 s

4. (d) To see the proper length of an object, an observer must be at rest with respect to the two points defining that length. Observers in either spacecraft see the other spacecraft as moving. Therefore, neither the observers in spacecraft A nor those in spacecraft B see the proper length of the other spacecraft.

5. 8.19 light-years

6. (b) The runner sees home plate move away from his feet and first base arrive at his feet. Thus, the runner sees both events occurring at the same place and measures the proper time. The catcher is the one at rest with respect to home plate and first base. Therefore, he measures the proper length between the two points.

7. (c) According to the theory of special relativity, the equations apply when both observers have constant velocities with respect to an inertial reference frame.

8. (c) The observers will always disagree about the time interval and the length, as indicated by the time-dilation and length-contraction equations. However, each will measure the same relative speed for the other’s motion.

9. 22.7 m

10. (c) The expression \( p = \frac{mv}{\sqrt{1 - v^2/c^2}} \) for the magnitude of the relativistic momentum applies at any speed \( v \). When it is used, the conservation of linear momentum is valid for an isolated system no matter what the speeds of the various parts of the system are.

11. (d) Both of the expressions can be used provided that \( v \ll c \). Expression B differs from expression A only by a negligible amount in this limit of small speeds.

12. 0.315 kg·m/s
13. (b) The mass $m$ of an object is proportional to the object’s rest energy $E_0$, according to $E_0 = mc^2$. The rest energy includes all forms of energy except kinetic energy, which plays no role here, because the glass is not moving. To freeze half the liquid into ice, energy in the form of heat must be removed from the liquid, so the water in possibility B has more mass than the water in possibility A. To freeze the remaining liquid into ice, more heat must be removed, so the water in possibility A has more mass than the water in possibility C. Thus, the ranking in descending order (largest first) is B, A, C.

14. $1.88 \times 10^{-5}$ kg

15. (b) The total energy is the sum of the rest energy and the kinetic energy. The rest energy includes all forms of energy (including potential energy) except kinetic energy.

16. $2.60 \times 10^8$ m/s

17. (a) According to Equation 28.7, the magnitude of the momentum is $p = \frac{\sqrt{E^2 - m^2 c^4}}{c}$, where $E$ is the total energy. The total energy is $E = E_0 + KE$. Since the kinetic energy is equal to the rest energy, the total energy is $E = 2E_0$. Substituting this result into the expression for $p$ and using the fact that $mc^2 = E_0$ give $p = \frac{\sqrt{4E_0^2 - E_0^2}}{c} = \frac{\sqrt{3mc^2}}{c}$.

18. $2.18 \times 10^8$ m/s
CHAPTER 28

SPECIAL RELATIVITY

PROBLEMS

1. **REASONING**
   a. The two events in this problem are the creation of the pion and its subsequent decay (or breaking apart). Imagine a reference frame attached to the pion, so the pion is stationary relative to this reference frame. To a hypothetical person who is at rest with respect to this reference frame, these two events occur at the same place, namely, at the place where the pion is located. Thus, this hypothetical person measures the proper time interval \( \Delta t_0 \) for the decay of the pion. On the other hand, the person standing in the laboratory sees the two events occurring at different locations, since the pion is moving relative to that person. The laboratory person, therefore, measures a dilated time interval \( \Delta t \). The relation between these two time intervals is given by 
   \[
   \Delta t = \Delta t_0 \sqrt{1 - \frac{v^2}{c^2}} \tag{Equation 28.1}
   \]
   (Equation 28.1).

   b. According to the hypothetical person who is at rest in the reference frame attached to the moving pion, the distance \( x \) that the laboratory travels before the pion breaks apart is equal to the speed \( v \) of the laboratory relative to the pion times the proper time interval \( \Delta t_0 \), or 
   \[
   x = v \Delta t_0 .
   \]
   The speed of the laboratory relative to the pion is the same as the speed of the pion relative to the laboratory, namely, \( 0.990c \).

**SOLUTION**
   a. The proper time interval is 
   \[
   \Delta t_0 = \Delta t \sqrt{1 - \frac{v^2}{c^2}} = \left(3.5 \times 10^{-8} \text{ s}\right) \sqrt{1 - \left(\frac{0.990c}{c}\right)^2} = 4.9 \times 10^{-9} \text{ s}
   \]

   b. The distance \( x \) that the laboratory travels before the pion breaks apart, as measured by the hypothetical person, is 
   \[
   x = v \Delta t_0 = \left(0.990\right)\left(3.00 \times 10^8 \text{ m/s}\right)\left(4.9 \times 10^{-9} \text{ s}\right) = 1.5 \text{ m}
   \]

2. **REASONING** The time interval \( \Delta t_0 = 25 \text{ s} \) measured on earth is the proper time interval. This is because an observer on earth makes his measurement by noting the time it takes for a spot on the antenna to move around a complete circle. Hence, such an observer is at rest with respect to this spot as it starts around the circle and finishes its rotation in the same place. The time interval \( \Delta t = 42 \text{ s} \) measured by instruments on the moving spacecraft is the dilated time interval. This is because the instruments “observe” the earth to be moving relative to the spaceship and a spot on the antenna to start and end its rotational path at different places. The two time intervals are related by the time-dilation equation
\[ \Delta t = \frac{\Delta t_0}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}} \] (Equation 28.1), where \( v \) is the speed of the spaceship with respect to the earth and \( c \) is the speed of light in a vacuum.

**SOLUTION** Solving the time dilation equation for \( \frac{v}{c} \), we find

\[
\Delta t = \frac{\Delta t_0}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}} \quad \text{or} \quad \sqrt{1 - \left(\frac{v^2}{c^2}\right)} = \frac{\Delta t_0}{\Delta t}
\]

or

\[
\left[1 - \left(\frac{v^2}{c^2}\right)\right] = \left(\frac{\Delta t_0}{\Delta t}\right)^2 \quad \text{or} \quad 1 - \left(\frac{v^2}{c^2}\right) = \left(\frac{\Delta t_0}{\Delta t}\right)^2
\]

or

\[
v^2 / c^2 = 1 - \left(\frac{\Delta t_0}{\Delta t}\right)^2 \quad \text{or} \quad \frac{v}{c} = \sqrt{1 - \left(1 - \left(\frac{\Delta t_0}{\Delta t}\right)^2\right)} = \sqrt{1 - \left(\frac{25 \text{ s}}{42 \text{ s}}\right)^2} = 0.80
\]

3. **SSM REASONING** The total time for the trip is one year. This time is the proper time interval \( \Delta t_0 \), because it is measured by an observer (the astronaut) who is at rest relative to the beginning and ending events (the times when the trip started and ended) and who sees them at the same location in spacecraft. On the other hand, the astronaut measures the clocks on earth to run at the dilated time interval \( \Delta t \), which is the time interval of one hundred years. The relation between the two time intervals is given by Equation 28.1, which can be used to find the speed of the spacecraft.

**SOLUTION** The dilated time interval \( \Delta t \) is related to the proper time interval \( \Delta t_0 \) by

\[
\Delta t = \frac{\Delta t_0}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}}
\]

Solving this equation for the speed \( v \) of the spacecraft yields

\[
v = c \sqrt{1 - \left(\frac{\Delta t_0}{\Delta t}\right)^2} = c \sqrt{1 - \left(\frac{1 \text{ yr}}{100 \text{ yr}}\right)^2} = 0.99995c \quad (28.1)
\]

4. **REASONING** When you measure your breathing rate, you count \( N = 8.0 \) breaths during a proper time interval of \( \Delta t_0 = 1.0 \) minutes, and in so doing you determine a rate of

\[
R_0 = \frac{N}{\Delta t_0} = \frac{8.0 \text{ breaths}}{1.0 \text{ minute}} = 8.0 \text{ breaths/minute}
\]

When measured by monitors on the earth, the \( N = 8.0 \) breaths occur in a dilated time interval \( \Delta t \) that is related to the proper time interval by

\[
\Delta t = \frac{\Delta t_0}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}}
\]

(Equation 28.1). The breathing rate \( R \) measured by a monitor on the earth, then, is given by
\[ R = \frac{N}{\Delta t} \]  

**SOLUTION** Substituting \( \Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \) (Equation 28.1) into Equation (1), we obtain

\[
R = \frac{N}{\Delta t} = \frac{N}{\Delta t_0} \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{N}{\Delta t_0} \frac{8.0 \text{ breaths}}{1.0 \text{ minute}} = 1.8 \text{ breaths/minute}
\]

5. **REASONING** The observer is moving with respect to the oscillating object. Therefore, to the observer, the oscillating object is moving with a speed of \( v = 1.90 \times 10^8 \text{ m/s} \), and the observer measures a dilated time interval for the period of oscillation. To determine this dilated time interval \( \Delta t = T_{\text{dilated}} \), we must use the time-dilation equation:

\[
\Delta t = T_{\text{dilated}} = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (28.1)
\]

where \( \Delta t_0 \) is the proper time interval, as measured in the reference frame to which the fixed end of the spring is attached. The proper time interval is the period \( T \) of the oscillation as given by Equations 10.4 and 10.11:

\[
\Delta t_0 = T = 2\pi \sqrt{\frac{m}{k}} \quad (1)
\]

where \( m \) is the mass of the object and \( k \) is the spring constant.

**SOLUTION** Substituting Equation (1) into the time-dilation equation gives

\[
T_{\text{dilated}} = \frac{2\pi \sqrt{\frac{m}{k}}}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{2\pi \sqrt{\frac{6.00 \text{ kg}}{76.0 \text{ N/m}}}}{\sqrt{1 - \frac{(1.90 \times 10^8 \text{ m/s})^2}{(3.00 \times 10^8 \text{ m/s})^2}}} = 2.28 \text{ s}
\]

6. **REASONING** The distance \( d \) traveled by the ship, according to an observer on earth, is equal to the product of the speed \( v \) of the ship relative to earth and the elapsed time \( \Delta t \) measured by the earthbound observer, according to Equation 2.1:

\[
d = v\Delta t \quad (2.1)
\]
The time interval $\Delta t$ is the dilated time interval and is related to the proper time interval $\Delta t_0$ for the journey (as measured by an observer on the ship) via Equation 28.1:

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{(28.1)}$$

In this equation, $v$ is the speed of the ship relative to earth, and $c$ is the speed of light in a vacuum. We will use Equation 28.1 to determine the speed $v$ of the ship, and then Equation 2.1 to find the distance the ship travels, according to the earthbound observer.

**SOLUTION**  Squaring both sides of Equation 28.1 and solving for the ratio $v^2/c^2$, we obtain

$$(\Delta t)^2 = \frac{(\Delta t_0)^2}{1 - \frac{v^2}{c^2}} \quad \text{or} \quad 1 - \frac{v^2}{c^2} = \frac{(\Delta t_0)^2}{(\Delta t)^2} \quad \text{or} \quad 1 - \left( \frac{\Delta t_0}{\Delta t} \right)^2 = \frac{v^2}{c^2} \quad \text{(1)}$$

Solving Equation (1) for $v$ yields

$$v^2 = c^2 \left[ 1 - \left( \frac{\Delta t_0}{\Delta t} \right)^2 \right] \quad \text{or} \quad v = c \sqrt{1 - \left( \frac{\Delta t_0}{\Delta t} \right)^2} \quad \text{(2)}$$

Substituting Equation (2) into Equation 2.1, we find that

$$d = v\Delta t = c \sqrt{1 - \left( \frac{\Delta t_0}{\Delta t} \right)^2} \Delta t$$

$$= \left(3.0 \times 10^8 \text{ m/s} \right) \sqrt{1 - \left( \frac{9.2 \text{ yr}}{24.0 \text{ h}} \right)^2} \left( \frac{3.16 \times 10^7 \text{ s}}{1 \text{ yr}} \right) = 7.3 \times 10^{16} \text{ m}$$

7. **REASONING AND SOLUTION**  The proper time is the time it takes for the bacteria to double its number, i.e., $\Delta t_0 = 24.0$ hours. For the earth based sample to grow to 256 bacteria, it would take 8 days ($2^n = 256$ or $n = 8$). The "doubling time" for the space culture would be

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \left( \frac{v}{c} \right)^2}} = \frac{24.0 \text{ h}}{\sqrt{1 - \left( \frac{0.866 c}{c} \right)^2}} = 48.0 \text{ h} \quad \text{or} \quad 2 \text{ days}$$

In eight earth days, the space bacteria would undergo $n' = \left( \frac{1}{2} \right)^8 = 4$ "doublings". The number of space bacteria is

$$\text{Number of Space Bacteria} = 2^n' = 2^4 = 16$$
8. **REASONING** The distance \( L_0 = 4.1 \times 10^6 \text{ m} \) is the proper distance, because it is the distance between the two cities as measured by an observer on the earth who is at rest with respect to the cities. The distance \( L \) measured by the voyagers aboard the UFO is the contracted distance, because the voyagers are moving relative to the cities. The two distances are related by the length-contraction equation \( L = L_0 \sqrt{1 - \left(\frac{v}{c}\right)^2} \) (Equation 28.2), where \( v = 0.70c \) is the speed of the UFO with respect to the earth and \( c \) is the speed of light in a vacuum.

**SOLUTION** Using Equation 28.2, we find that the contracted distance measured by the UFO-voyagers is

\[
L = L_0 \sqrt{1 - \left(\frac{v}{c}\right)^2} = \left(4.1 \times 10^6 \text{ m}\right) \sqrt{1 - \left(\frac{0.70c}{c}\right)^2} = 2.9 \times 10^6 \text{ m}
\]

9. **SSM REASONING** All standard meter sticks at rest have a length of 1.00 m for observers who are at rest with respect to them. Thus, 1.00 m is the proper length \( L_0 \) of the meter stick. When the meter stick moves with speed \( v \) relative to an earth-observer, its length \( L = 0.500 \text{ m} \) will be a contracted length. Since both \( L_0 \) and \( L \) are known, \( v \) can be found directly from Equation 28.2, \( L = L_0 \sqrt{1 - \left(\frac{v^2}{c^2}\right)} \).

**SOLUTION** Solving Equation 28.2 for \( v \), we find that

\[
v = c \sqrt{1 - \left(\frac{L}{L_0}\right)^2} = \left(3.00 \times 10^8 \text{ m/s}\right) \sqrt{1 - \left(\frac{0.500 \text{ m}}{1.00 \text{ m}}\right)^2} = 2.60 \times 10^8 \text{ m/s}
\]

10. **REASONING** The distance between earth and the center of the galaxy is the proper length \( L_0 \), because it is the distance measured by an observer who is at rest relative to the earth and the center of the galaxy. A person on board the spaceship is moving with respect to them and measures a contracted length \( L \) that is related to the proper length by Equation 28.2 as

\[
L = L_0 \sqrt{1 - \left(\frac{v^2}{c^2}\right)}.
\]

The contracted distance is also equal to the product of the spaceship’s speed \( v \) the time interval measured by a person on board the spaceship. This time interval is the proper time interval \( \Delta t_0 \) because the person on board the spaceship measures the beginning and ending events (the times when the trip starts and ends) at the same location relative to a coordinate system fixed to the spaceship. Thus, the contracted distance is also \( L = v \Delta t_0 \). By setting the two expressions for \( L \) equal to each other, we can find the how long the trip will take according to a clock on board the spaceship.
**SOLUTION** Setting \( L = L_0 \sqrt{1 - \left( \frac{v^2}{c^2} \right)} \) equal to \( L = v \Delta t_0 \) and solving for the proper time interval \( \Delta t_0 \) gives

\[
\Delta t_0 = \frac{L_0}{v} \sqrt{1 - \left( \frac{v^2}{c^2} \right)}
\]

\[
= \frac{(23,000 \text{ ly}) \left( \frac{9.47 \times 10^{15} \text{ m}}{1 \text{ ly}} \right)}{0.9990 \left( 3.00 \times 10^8 \text{ m/s} \right)} \sqrt{1 - \left( \frac{(0.9990c)^2}{c^2} \right)} \left( \frac{1 \text{ yr}}{3.16 \times 10^7 \text{ s}} \right) = 1.0 \times 10^3 \text{ yr}
\]

11. **REASONING** The tourist is moving at a speed of \( v = 1.3 \text{ m/s} \) with respect to the path and, therefore, measures a contracted length \( L \) instead of the proper length of \( L_0 = 9.0 \text{ km} \). The contracted length is given by the length-contraction equation, Equation 28.2.

**SOLUTION** According to the length-contraction equation, the tourist measures a length that is

\[
L = L_0 \sqrt{1 - \left( \frac{v^2}{c^2} \right)} = (9.0 \text{ km}) \sqrt{1 - \left( \frac{1.3 \text{ m/s}}{3.0 \text{ m/s}} \right)^2} = 8.1 \text{ km}
\]

12. **REASONING** The Martian measures the proper time interval \( \Delta t_0 \), because the Martian measures the beginning and ending events (the times when the trip starts and ends) at the same location relative to a coordinate system fixed to the spaceship.

The given distance between Mars and Venus is the distance as measured by a person on earth. That person is at rest relative to the two planets and, hence, measures the proper length. The Martian, who is moving relative to the planets, does not measure the proper length, but measures a contracted length.

According to the Martian, the time of the trip \( \Delta t_0 \) is equal to the contracted length that he measures divided by the speed \( v \) of the spaceship.

**SOLUTION**

a. The contracted length \( L \) measured by the Martian is related to the proper length \( L_0 \) by Equation 28.2 as

\[
L = L_0 \sqrt{1 - \left( \frac{v^2}{c^2} \right)} = (1.20 \times 10^{11} \text{ m}) \sqrt{1 - \left( \frac{0.80c}{c} \right)^2} = 7.2 \times 10^{10} \text{ m}
\]
b. The time of the trip as measured by the Martian is

\[
\Delta t_0 = \frac{L}{v} = \frac{7.2 \times 10^3 \text{ m}}{0.80 \left(3.00 \times 10^8 \text{ m/s}\right)} = 3.0 \times 10^2 \text{ s}
\]

13. **REASONING AND SOLUTION** The diameter \(D\) of the planet, as measured by a moving spacecraft, is given in terms of the proper diameter \(D_0\) by Equation 28.2. Taking the ratio of the diameter \(D_A\) of the planet measured by spaceship A to the diameter \(D_B\) measured by spaceship B, we find

\[
\frac{D_A}{D_B} = \frac{D_0 \sqrt{1 - \frac{v_A^2}{c^2}}}{D_0 \sqrt{1 - \frac{v_B^2}{c^2}}} = \frac{\sqrt{1 - (0.60 c)^2}}{\sqrt{1 - (0.80 c)^2}} = 1.3
\]

14. **REASONING**

a. The two events are the creation of the particle and its subsequent disintegration. Relative to a stationary reference frame fixed to the laboratory, these two events occur at different locations, because the particle is moving relative to this reference frame. The proper distance \(L_0\) is the distance \((1.05 \times 10^{-3} \text{ m})\) given in the statement of the problem, because this distance is measured by an observer in the laboratory who is at rest with respect to these locations.

b. The distance measured by a hypothetical person traveling with the particle is a contracted distance, because it is measured by a person who is moving relative to the two locations. The contracted distance \(L\) is related to the proper distance \(L_0\) by the length-contraction formula, \(L = L_0 \sqrt{1 - v^2 / c^2}\) (Equation 28.2).

c. The proper lifetime \(\Delta t_0\) of the particle is the lifetime as registered in a reference frame attached to the particle. In this reference frame the two events occur at the same location. The proper lifetime is equal to the contracted distance \(L\), which is measured in this reference frame, divided by the speed \(v\) of the particle, or \(\Delta t_0 = L/v\).

d. The particle’s contracted lifetime \(\Delta t\) is related to its proper lifetime \(\Delta t_0\) by the time-dilation formula, \(\Delta t = \Delta t_0 / \sqrt{1 - v^2 / c^2}\) (Equation 28.1).

**SOLUTION**

a. The proper distance is \(L_0 = 1.05 \times 10^{-3} \text{ m}\).

b. The distance measured by a hypothetical person traveling with the particle is
\[ L = L_0 \sqrt{1 - \frac{v^2}{c^2}} = \left(1.05 \times 10^{-3} \text{ m}\right) \sqrt{1 - \left(\frac{0.990c}{c}\right)^2} = 1.48 \times 10^{-4} \text{ m} \]  

(28.2)

c. The proper lifetime \( \Delta t_0 \) is equal to the contracted distance \( L \) divided by the speed \( v \) of the particle:

\[ \Delta t_0 = \frac{L}{v} = \frac{1.48 \times 10^{-4} \text{ m}}{\left(0.990 \times 3.00 \times 10^8 \text{ m/s}\right) / 0.990c} = 4.98 \times 10^{-13} \text{ s} \]

d. The dilated lifetime is

\[ \Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{\left(4.98 \times 10^{-13} \text{ s}\right)}{\sqrt{1 - \left(\frac{0.990c}{c}\right)^2}} = 3.53 \times 10^{-12} \text{ s} \]  

(28.1)

15. **REASONING** Length contraction occurs only along the direction of the motion. Those dimensions that are perpendicular to the motion are not contracted. In this problem, then, we expect side \( x_0 \) to be contracted to a value of \( x \), whereas side \( y_0 \), being perpendicular, will be unaffected by the motion. It is because of the contraction of side \( x_0 \) that the person aboard the rocket will measure an angle \( \theta \) that is different than 30.0\(^\circ\). To determine \( \theta \), we will use the fact that the tangent of an angle in a right triangle is the opposite side divided by the adjacent side (see Equation 1.3) of the triangle. To take into account the length contraction, we will use the length-contraction equation \( x = x_0 \sqrt{1 - \left(\frac{v}{c}\right)^2} \) (Equation 28.2), where \( v = 0.70c \) is the speed of the rocket with respect to the space station and \( c \) is the speed of light in a vacuum.

**SOLUTION** Since side \( x_0 \) is contracted to a value of \( x \) while side \( y_0 \) is unaffected by the motion and since the tangent of an angle in a right triangle is the opposite side divided by the adjacent side (see Equation 1.3), we know that

\[ \tan \theta = \frac{y_0}{x} \]  

(1)

In Equation (1) the contracted length \( x \) is related to the length \( x_0 \) by the length-contraction equation \( x = x_0 \sqrt{1 - \left(\frac{v}{c}\right)^2} \) (Equation 28.2). Substituting this expression for \( x \) into Equation (1) gives

\[ \tan \theta = \frac{y_0}{x} = \frac{y_0}{x_0 \sqrt{1 - \left(\frac{v}{c}\right)^2}} \]  

(2)

Note that \( \tan 30.0^\circ = \frac{y_0}{x_0} \), so that Equation (2) becomes
\[ \tan \theta = \frac{y_0}{x_0 \sqrt{1 - \left( \frac{v}{c} \right)^2}} = \tan 30.0^\circ = \tan 30.0^\circ = 0.845 \]

Thus, we find that \[ \theta = \tan^{-1} 0.845 = 40.2^\circ \]

16. **REASONING** To the observer at rest relative to the cube, its length, width, and height are all equal to the proper length \( L_0 = 0.11 \) m of one of the cube’s sides. Suppose that the moving observer is moving parallel to the width of the cube and, therefore, measures a contracted length \( L \) for the width. Note, however, that this observer still measures the proper length \( L_0 \) for the other two dimensions of the cube, since they are perpendicular to the direction of motion. The shortened width of the cube is given by \( L = L_0 \sqrt{1 - \frac{v^2}{c^2}} \) (Equation 28.2), where \( v \) is the speed of the observer relative to the cube. Accordingly, the volume \( V = L_0 \times L \times L_0 = L_0^2 L \) of the cube is smaller for the moving observer than it is for the observer at rest relative to the cube. Given the mass \( m \) of the cube, then, the moving observer calculates a density of \( \rho = \frac{m}{V} = \frac{m}{L_0^2 L} \) (Equation 11.1) for the cube that is greater than the density of glass.

**SOLUTION** Substituting \( L = L_0 \sqrt{1 - \frac{v^2}{c^2}} \) (Equation 28.2) into \( \rho = \frac{m}{L_0^2 L} \) (Equation 11.1), we obtain

\[ \rho = \frac{m}{L_0^2 L} = \frac{m}{(L_0^2) L_0 \sqrt{1 - \frac{v^2}{c^2}}} = \frac{m}{L_0^3 \sqrt{1 - \frac{v^2}{c^2}}} \]

Rearranging Equation (1) and solving for the quantity \( \frac{v^2}{c^2} \), we find that

\[ \frac{m}{\rho L_0^3} = \sqrt{1 - \frac{v^2}{c^2}} \quad \text{or} \quad \left( \frac{m}{\rho L_0^3} \right)^2 = 1 - \frac{v^2}{c^2} \quad \text{or} \quad \frac{v^2}{c^2} = 1 - \left( \frac{m}{\rho L_0^3} \right)^2 \]

Taking the square root of both sides of Equation (2) and solving for \( v \) yields

\[ \frac{v}{c} = \sqrt{1 - \left( \frac{m}{\rho L_0^3} \right)^2} \quad \text{or} \quad v = c \sqrt{1 - \left( \frac{m}{\rho L_0^3} \right)^2} = c \sqrt{1 - \left[ \frac{3.2 \text{ kg}}{(7800 \text{ kg/m}^3)(0.11 \text{ m})^3} \right]^2} = 0.951c \]
17. **REASONING** Only the sides of the rectangle that lie in the direction of motion will experience length contraction. In order to make the rectangle look like a square, each side must have a length of $L = 2.0 \text{ m}$. Thus, we move along the long side, taking the proper length to be $L_0 = 3.0 \text{ m}$. We can solve for the speed using Equation 28.2. Then, with this speed, we can use the relation for length contraction to find $L$ for the short side as we move along it.

**SOLUTION** From Equation 28.2, $L = L_0 \sqrt{1 - \left(\frac{v^2}{c^2}\right)}$, we find that

$$v = c \sqrt{1 - \left(\frac{L}{L_0}\right)^2} = c \sqrt{1 - \left(\frac{2.0 \text{ m}}{3.0 \text{ m}}\right)^2} = 0.75 c$$

Moving at this speed along the short side, we take $L_0 = 2.0 \text{ m}$ and find $L$:

$$L = L_0 \sqrt{1 - \left(\frac{v}{c}\right)^2} = (2.0 \text{ m}) \sqrt{1 - \left(\frac{0.75 c}{c}\right)^2} = 1.3 \text{ m}$$

The observed dimensions of the rectangle are, therefore, $3.0 \text{ m} \times 1.3 \text{ m}$, since the long side is not contracted due to motion along the short side.

18. **REASONING** The magnitude $p$ of the relativistic momentum of a particle (mass $m$ and speed $v$) is given by Equation 28.3:

$$p = \frac{mv}{\sqrt{1 - v^2 / c^2}} = \frac{p_0}{\sqrt{1 - v^2 / c^2}}$$

where we have used the fact that the magnitude $p_0$ of the nonrelativistic momentum of a particle is $p_0 = mv$ (Equation 7.2).

**SOLUTION** When the magnitude of the relativistic momentum of a particle is three times the magnitude of its nonrelativistic momentum, we have $p = 3p_0$, so that Equation (1) becomes

$$3p_0 = \frac{p_0}{\sqrt{1 - v^2 / c^2}} \quad \text{or} \quad \sqrt{1 - v^2 / c^2} = \frac{1}{3}$$

Squaring and rearranging the last expression gives

$$1 - \frac{v^2}{c^2} = \frac{1}{9} \quad \text{or} \quad \frac{v^2}{c^2} = 1 - \frac{1}{9} = \frac{8}{9}$$

Taking the square root, we find

$$v = \sqrt{\frac{8}{9}} c = 0.943 c = 2.83 \times 10^8 \text{ m/s}$$
19. **REASONING** The magnitude of the relativistic momentum $p$ of the proton is related to its relativistic total energy $E$ by $E^2 = p^2c^2 + m^2c^4$ (Equation 28.7), where $m = 1.67 \times 10^{-27}$ kg is the mass of a proton (see the inside front cover of the text) and $c$ is the speed of light in a vacuum.

**SOLUTION** Solving Equation 28.7 for $p$, we obtain

$$p^2c^2 = E^2 - m^2c^4 \quad \text{or} \quad p^2 = \frac{E^2}{c^2} - m^2c^2 \quad \text{or} \quad p = \sqrt{\frac{E^2}{c^2} - m^2c^2}$$

Therefore, the magnitude of the relativistic momentum of the proton is

$$p = \sqrt{\left(\frac{2.7 \times 10^{-10} \text{ J}}{3.00 \times 10^8 \text{ m/s}}\right)^2 - \left(1.67 \times 10^{-27} \text{ kg}\right)^2 \left(3.00 \times 10^8 \text{ m/s}\right)^2} = 7.5 \times 10^{-19} \text{ kg} \cdot \text{m/s}$$

20. **REASONING** The magnitude $p_{rel}$ of the relativistic momentum of the spacecraft is given by $p_{rel} = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}}$ (Equation 28.3), where $m$ is the mass of the spacecraft, $v$ is its speed, and $c$ is the speed of light in a vacuum. The numerator of Equation 28.3 is the magnitude of the nonrelativistic momentum $p = mv$ (Equation 7.2), so we have that

$$p_{rel} = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{p}{\sqrt{1 - \frac{v^2}{c^2}}}$$

We are told that the pilot measures the proper time interval $\Delta t_0$ between two events to be half the dilated time interval $\Delta t$: $\Delta t_0 = \frac{1}{2} \Delta t$. Thus, the dilated time interval is

$$\Delta t = 2\Delta t_0 \quad (2)$$

In general, the proper time interval $\Delta t_0$ and the dilated time interval $\Delta t$ are related by

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (28.1)$$

**SOLUTION** Substituting Equation (2) into Equation 28.1, and solving for the expression $\sqrt{1 - \frac{v^2}{c^2}}$ that also appears in Equation (1), we obtain
\[ \Delta t = 2 \Delta \gamma = \frac{\Delta \gamma}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{or} \quad 2 = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{or} \quad \sqrt{1 - \frac{v^2}{c^2}} = \frac{1}{2} \quad (3) \]

Substituting Equation (3) into Equation (1), we find that

\[ p_{\text{rel}} = \frac{p}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{p}{\left(\frac{1}{2}\right)} = 2p = 2 \left(1.3 \times 10^{13} \text{ kg} \cdot \text{m/s}\right) = 2.6 \times 10^{13} \text{ kg} \cdot \text{m/s} \]

21. **REASONING** The height of the woman as measured by the observer is given by Equation 28.2 as \( h = h_0 \sqrt{1 - \left(v/c\right)^2} \), where \( h_0 \) is her proper height. In order to use this equation, we must determine the speed \( v \) of the woman relative to the observer. We are given the magnitude of her relativistic momentum, so we can determine \( v \) from \( p \).

**SOLUTION** According to Equation 28.3 \( p = mv \sqrt{1 - v^2/c^2} \), so \( mv = p \sqrt{1 - v^2/c^2} \). Squaring both sides, we have

\[ m^2v^2 = p^2(1 - v^2/c^2) \quad \text{or} \quad m^2v^2 + p^2 \frac{v^2}{c^2} = p^2 \]

\[ v^2 \left( m^2 + \frac{p^2}{c^2} \right) = p^2 \quad \text{or} \quad v^2 = \frac{p^2}{m^2 + \frac{p^2}{c^2}} \]

Solving for \( v \) and substituting values, we have

\[ v = \frac{p}{\sqrt{m^2 + \frac{p^2}{c^2}}} = \frac{2.0 \times 10^{10} \text{ kg} \cdot \text{m/s}}{\sqrt{(55 \text{ kg})^2 + \left(\frac{2.0 \times 10^{10} \text{ kg} \cdot \text{m/s}}{3.0 \times 10^8 \text{ m/s}}\right)^2}} = 2.3 \times 10^8 \text{ m/s} \]

Then, the height that the observer measures for the woman is

\[ h = h_0 \sqrt{1 - \left(\frac{v}{c}\right)^2} = (1.6 \text{ m}) \sqrt{1 - \left(\frac{2.3 \times 10^8 \text{ m/s}}{3.0 \times 10^8 \text{ m/s}}\right)^2} = 1.0 \text{ m} \]

22. **REASONING** In special relativity the momentum of a particle is given by Equation 28.3 as \( p = mv \sqrt{1 - \left(v^2/c^2\right)} \). Because of the \( \sqrt{1 - \left(v^2/c^2\right)} \) term in the denominator, doubling the particle’s speed more than doubles its momentum.
An examination of Equation 28.3 shows that the relativistic momentum is directly proportional to the mass $m$. Thus, halving the particle’s mass also halves its momentum.

The relation $p = \frac{mv}{\sqrt{1 - v^2 / c^2}}$ indicates that the magnitudes of the relativistic momenta for particles a, b, and c are:

- Particle a: $p_a = \frac{mv}{\sqrt{1 - v^2 / c^2}} = \frac{(1.20 \times 10^{-8} \text{ kg})(0.200)(3.00 \times 10^8 \text{ m/s})}{\sqrt{1 - (2 \times 0.200\text{c})^2}} = 0.735 \text{ kg} \cdot \text{m/s}$

- Particle b: $p_b = \frac{mv}{\sqrt{1 - v^2 / c^2}} = \frac{(1.20 \times 10^{-8} \text{ kg})(2 \times 0.200)(3.00 \times 10^8 \text{ m/s})}{\sqrt{1 - (4 \times 0.200\text{c})^2}} = 0.786 \text{ kg} \cdot \text{m/s}$

- Particle c: $p_c = \frac{mv}{\sqrt{1 - v^2 / c^2}} = \frac{(1.20 \times 10^{-8} \text{ kg})(4 \times 0.200)(3.00 \times 10^8 \text{ m/s})}{\sqrt{1 - (16 \times 0.200\text{c})^2}} = 1.20 \text{ kg} \cdot \text{m/s}$

These results show that particle c has the greatest momentum magnitude, followed by particle b and then by particle a.

**SOLUTION** The momenta of the three particles are:

- Particle a: $p_a = \frac{mv}{\sqrt{1 - v^2 / c^2}} = \frac{(1.20 \times 10^{-8} \text{ kg})(0.200)(3.00 \times 10^8 \text{ m/s})}{\sqrt{1 - (2 \times 0.200\text{c})^2}} = 0.735 \text{ kg} \cdot \text{m/s}$

- Particle b: $p_b = \frac{mv}{\sqrt{1 - v^2 / c^2}} = \frac{(1.20 \times 10^{-8} \text{ kg})(2 \times 0.200)(3.00 \times 10^8 \text{ m/s})}{\sqrt{1 - (4 \times 0.200\text{c})^2}} = 0.786 \text{ kg} \cdot \text{m/s}$

- Particle c: $p_c = \frac{mv}{\sqrt{1 - v^2 / c^2}} = \frac{(1.20 \times 10^{-8} \text{ kg})(4 \times 0.200)(3.00 \times 10^8 \text{ m/s})}{\sqrt{1 - (16 \times 0.200\text{c})^2}} = 1.20 \text{ kg} \cdot \text{m/s}$

As expected, the ranking of the momenta (largest first) is c, b, a.

23. **SSM REASONING** The magnitude $p$ of the relativistic momentum of an object is given by $p = \frac{mv}{\sqrt{1 - v^2 / c^2}}$ (Equation 28.3), where $m$ is the object’s mass, $v$ is the object’s speed, and $c$ is the speed of light in a vacuum. The principle of conservation of linear momentum (see Section 7.2) states that the total momentum of a system is conserved when no net external force acts on the system. This principle applies at speeds approaching the speed of light in a vacuum, provided that Equation 28.3 is used for the individual momenta of the objects that comprise the system.
**SOLUTION** The total momentum of the man/woman system is conserved, since friction is negligible, so that no net external force acts on the system. Therefore, the final total momentum \( p_m + p_w \) must equal the initial total momentum, which is zero. As a result, \( p_m = -p_w \) where Equation 28.3 must be used for the momenta \( p_m \) and \( p_w \). Thus, we find

\[
\frac{m_m v_m}{\sqrt{1-(v_m/c)^2}} = -\frac{m_w v_w}{\sqrt{1-(v_w/c)^2}} \tag{1}
\]

We know that \( m_m = 88 \text{ kg}, \ m_w = 54 \text{ kg}, \) and \( v_w = +2.5 \text{ m/s}. \) Remember that \( c \) has the hypothetical value of \( 3.0 \text{ m/s}. \) Solving Equation (1) for \( v_m \) reveals that \( v_m = \pm 2.0 \text{ m/s}. \) We choose the negative value, since the man and woman recoil from one another and it is stated that the woman moves away in the positive direction. Therefore, we find that \( v_m = -2.0 \text{ m/s}. \)

24. **REASONING AND SOLUTION** The mass equivalent is given by \( E_0 = KE = mc^2 \) or

\[
m = \frac{KE}{c^2} = \frac{7.8 \times 10^{-13} \text{ J}}{(3.00 \times 10^8 \text{ m/s})^2} = 8.7 \times 10^{-30} \text{ kg} \]

25. **SSM REASONING** According to the work-energy theorem, Equation 6.3, the work that must be done on the electron to accelerate it from rest to a speed of \( 0.990c \) is equal to the kinetic energy of the electron when it is moving at \( 0.990c. \)

**SOLUTION** Using Equation 28.6, we find that

\[
KE = mc^2 \left(1 - \frac{1}{\sqrt{1-(v/c)^2}}\right)
\]

\[
= (9.11 \times 10^{-31} \text{ kg})(3.00 \times 10^8 \text{ m/s})^2 \left(\frac{1}{\sqrt{1-(0.990c)^2 / c^2}} - 1\right) = 5.0 \times 10^{-13} \text{ J}
\]

26. **REASONING** Compressing the spring stores elastic potential energy, which increases the total energy of the spring. Because the spring is not in motion, it is the spring’s rest energy \( E_0 \) that increases, as well as its mass \( m \), according to \( E_0 = mc^2 \) (Equation 28.5). We will use the hypothetical value of \( c = 3.00 \times 10^2 \text{ m/s} \) for the speed of light in a vacuum in Equation...
28.5. The increase of $\Delta m = 0.010 \, \text{g} = 0.010 \times 10^{-3} \, \text{kg}$ in the mass of the spring, therefore, corresponds to an increase $\Delta E_0$ in the rest energy of the spring, where

$$\Delta E_0 = (\Delta m)c^2$$

(1)

The increase of the rest energy is equal to the spring’s elastic potential energy $PE_{\text{elastic}} = \frac{1}{2}kx^2$ (Equation 10.13), where $k$ is the spring constant of the spring, and $x$ is the distance by which the spring is compressed from its equilibrium length. Therefore, we have that

$$\Delta E_0 = PE_{\text{elastic}} = \frac{1}{2}kx^2$$

(2)

**SOLUTION** Setting the right sides of Equations (1) and (2) equal to one another and solving for $x^2$ yields

$$\Delta E_0 = \frac{1}{2}kx^2 = (\Delta m)c^2 \quad \text{or} \quad x^2 = \frac{2(\Delta m)c^2}{k}$$

(3)

Taking the square root of Equation (3), we obtain

$$x = \sqrt{\frac{2(\Delta m)c^2}{k}} = c\sqrt{\frac{2(\Delta m)}{k}} = \left(3.00 \times 10^2 \, \text{m/s}\right)\sqrt{\frac{2 \left(0.010 \times 10^{-3} \, \text{kg}\right)}{850 \, \text{N/m}}} = 0.046 \, \text{m}$$

27. **REASONING** The mass $m$ of the aspirin is related to its rest energy $E_0$ by Equation 28.5, $E_0 = mc^2$. Since it requires $1.1 \times 10^8 \, \text{J}$ to operate the car for twenty miles, we can calculate the number of miles that the car can go on the energy that is equivalent to the mass of one tablet.

**SOLUTION** We begin by converting the mass $m$ from milligrams (mg) to kilograms (kg):

$$(325 \, \text{mg})\left(\frac{1 \, \text{g}}{1000 \, \text{mg}}\right)\left(\frac{1 \, \text{kg}}{1000 \, \text{g}}\right) = 325 \times 10^{-6} \, \text{kg}$$

The number $N$ of miles the car can go on one aspirin tablet is

$$N = \frac{E_0}{(1.1 \times 10^8 \, \text{J})/(20.0 \, \text{mi})} = \frac{mc^2}{(1.1 \times 10^8 \, \text{J})/(20.0 \, \text{mi})}$$

$$= \frac{\left(325 \times 10^{-6} \, \text{kg}\right)\left(3.0 \times 10^8 \, \text{m/s}\right)^2}{(1.1 \times 10^8 \, \text{J})/(20.0 \, \text{mi})} = 5.3 \times 10^6 \, \text{mi}$$

28. **REASONING** In order to change a certain mass of ice at 0 °C into liquid water at 0 °C heat must be added, and heat is a form of energy. Therefore, the energy of the liquid water is
greater than that of the ice. According to special relativity, energy and mass are equivalent. Since the liquid water has the greater energy, it also has the greater mass.

Heat must also be added to boil water into steam. Following the same type of reasoning as in the case of melting, we conclude that the steam has the greater mass.

The amount of heat $Q$ that must be supplied to change the phase of $m$ kilograms of a substance is given by Equation 12.5 as $Q = mL$, where $L$ is the latent heat of the substance. Since the latent heat of vaporization $L_v$ for water is greater than the latent heat of fusion $L_f$ (see Table 12.3), the change in mass is greater when liquid water turns into steam at $100 \, ^\circ C$ than when ice turns into liquid water at $0 \, ^\circ C$.

**SOLUTION** The change in mass $\Delta m$ associated with a change in rest energy $\Delta E_0$ is given by Equation 28.5 as $\Delta m = \Delta E_0/c^2$. The change in rest energy is the heat $Q$ that must be added to change the phase of the water, so that $Q = mL$. Thus, the change in mass is $\Delta m = Q/c^2 = mL/c^2$.

a. According to Table 12.3, the latent heat of fusion for water is $L_f = 3.35 \times 10^5 \, J/kg$. The change in mass associated with the ice to liquid-water phase change at $0 \, ^\circ C$ is then,

$$\Delta m = \frac{mL_f}{c^2} = \frac{(2.00 \, \text{kg})(3.35 \times 10^5 \, \text{J/kg})}{(3.00 \times 10^8 \, \text{m/s})^2} = 7.44 \times 10^{-12} \, \text{kg}$$

b. According to Table 12.3, the latent heat of vaporization for water is $L_f = 2.26 \times 10^6 \, J/kg$. The change in mass associated with the liquid-water to steam phase change at $100 \, ^\circ C$ is

$$\Delta m = \frac{mL_v}{c^2} = \frac{(2.00 \, \text{kg})(2.26 \times 10^6 \, \text{J/kg})}{(3.00 \times 10^8 \, \text{m/s})^2} = 5.02 \times 10^{-11} \, \text{kg}$$

As expected, the change in mass for the liquid-to-steam phase change is greater than that for the ice-to-liquid phase change.

---

29. **SSM REASONING AND SOLUTION**

a. In Section 28.6 it is shown that when the speed of a particle is $0.01c$ (or less), the relativistic kinetic energy becomes nearly equal to the nonrelativistic kinetic energy. Since the speed of the particle here is $0.001c$, the ratio of the relativistic kinetic energy to the nonrelativistic kinetic energy is $1.0$.

b. Taking the ratio of the relativistic kinetic energy, Equation 28.6, to the nonrelativistic kinetic energy, $\frac{1}{2} mv^2$, we find that
\[
mc^2 \left( \frac{1}{\sqrt{1-(v^2/c^2)}} - 1 \right) = \frac{1}{\frac{1}{2}mv^2} 
\]

\[
= 2 \left( \frac{c}{v} \right)^2 \left( \frac{1}{\sqrt{1-(v^2/c^2)}} - 1 \right) 
\]

\[
= 2 \left( \frac{c}{0.970c} \right)^2 \left( \frac{1}{\sqrt{1-(0.970c)^2/c^2}} - 1 \right) = 6.6 
\]

30. **REASONING AND SOLUTION** The energy \( E_0 \) produced in one year is the product of the power \( P \) generated and the time \( t \), \( E_0 = Pt \). This energy is equivalent to an amount of mass \( m \) given by Equation 28.5 as \( E_0 = mc^2 \). Thus, we have that

\[
mc^2 = Pt \quad \text{or} \quad m = \frac{Pt}{c^2} 
\]

The mass of nuclear fuel consumed in one year (3.15 \( \times \) 10\(^7\) s) is

\[
m = \frac{Pt}{c^2} = \frac{(3.0 \times 10^9 \text{ W})(3.15 \times 10^7 \text{ s})}{(3.00 \times 10^8 \text{ m/s})^2} = 1.1 \text{ kg}
\]

31. **REASONING** The rate \( R \) that the quasar is losing mass is equal to the mass \( m \) it loses divided by the time \( t \) during which the loss occurs, or \( R = m/t \). The mass that the quasar loses is equivalent to a certain amount of energy \( E_0 \); this equivalency is expressed by \( m = E_0/c^2 \) (Equation 28.5), where \( c \) is the speed of light. According to Equation 6.10b, the energy radiated is equal to the average power \( \bar{P} \) times the time, \( E_0 = \bar{P}t \). The average power is the rate at which the quasar radiates energy.

**SOLUTION** Combining \( R = m/t \) and \( m = E_0/c^2 \), we find that

\[
R = \frac{m}{t} = \frac{c^2}{c^2} = \frac{E_0}{c^2t} 
\]

Since the energy radiated by the quasar is \( E_0 = \bar{P}t \), the rate at which the quasar loses mass can be written as

\[
R = \frac{1}{c^2} \left( \frac{E_0}{t} \right) = \frac{1}{c^2} \left( \frac{\bar{P}t}{t} \right) = \frac{\bar{P}}{c^2} = \frac{1.0 \times 10^{41} \text{ W}}{(3.00 \times 10^8 \text{ m/s})^2} = 1.1 \times 10^{24} \text{ kg/s}
\]
32. **REASONING** According to the work-energy theorem (see Section 6.2), the change in the kinetic energy equals the work $W$ done to accelerate the electron. Since the electron starts from rest, its initial kinetic energy is zero, so its final kinetic energy $KE$ equals the work done. This conclusion is valid for either nonrelativistic or relativistic speeds. The given potential difference has a magnitude of $|\Delta V| = 2.40 \times 10^7 \text{ V}$ and is related to the work done according to $|\Delta V| = W / |q_0|$ (Equation 19.4 without the minus sign), where $|q_0| = 1.60 \times 10^{-19} \text{ C}$ is the magnitude of the charge of the electron. The relativistic kinetic energy of the electron is $KE = mc^2\left(\frac{1}{\sqrt{1-v^2/c^2}} - 1\right)$ (Equation 28.6), where $m = 9.11 \times 10^{-31} \text{ kg}$ is the electron’s mass, $v$ is the electron’s speed, and $c$ is the speed of light in a vacuum.

**SOLUTION**

a. Recognizing that the electron’s final kinetic energy $KE$ equals the work $W$ done to accelerate the electron, we have

$$KE = W$$  \hspace{1cm} (1)

Using Equation 19.4 to obtain the work, we have

$$|\Delta V| = \frac{W}{|q_0|} \quad \text{or} \quad W = |\Delta V||q_0|$$  \hspace{1cm} (2)

Substituting Equation (2) into Equation (1), we find that the relativistic kinetic energy of the electron is

$$KE = W = |\Delta V||q_0| = \left(2.40 \times 10^7 \text{ V}\right)\left(1.60 \times 10^{-19} \text{ C}\right) = 3.84 \times 10^{-12} \text{ J}$$

b. Solving Equation 28.6 for the speed $v$, we proceed as follows:

$$KE = mc^2\left(\frac{1}{\sqrt{1-v^2/c^2}} - 1\right) \quad \text{or} \quad \frac{1}{\sqrt{1-v^2/c^2}} = \frac{KE}{mc^2} + 1$$

or

$$1-v^2/c^2 = \left[\frac{1}{(KE)/(mc^2) + 1}\right]^2 \quad \text{or} \quad v = c\sqrt{1 - \left[\frac{1}{(KE)/(mc^2) + 1}\right]^2}$$

or

$$v = c\sqrt{1 - \left[\frac{1}{\left(3.84 \times 10^{-12} \text{ J}\right)\left(9.11 \times 10^{-31} \text{ kg}\right)\left(3.00 \times 10^8 \text{ m/s}\right)^2 + 1}\right]^2} = 0.999781c$$
33. **REASONING** The total energy $E$ and the magnitude $p$ of the relativistic momentum are related according to Equation 28.7:

$$E^2 = p^2 c^2 + m^2 c^4 \quad \text{or} \quad p^2 = \frac{E^2 - m^2 c^4}{c^2} \quad (28.7)$$

We are given a value for the total energy, but do not have a value for the mass $m$. However, we recognize that the rest energy is $E_0 = mc^2$ (Equation 28.5). With this substitution, Equation 28.7 becomes

$$p^2 = \frac{E^2 - m^2 c^4}{c^2} = \frac{E^2 - E_0^2}{c^2}$$

We can obtain a value for the rest energy, because the total energy is the sum of the kinetic energy and the rest energy or $E = KE + E_0$. In other words, the rest energy is $E_0 = E - KE$, and we have values for both $E$ and $KE$. Using this substitution for the rest energy, our expression for $p^2$ becomes

$$p^2 = \frac{E^2 - E_0^2}{c^2} = \frac{E^2 - (E - KE)^2}{c^2} \quad (1)$$

**SOLUTION** Using Equation (1), we find that

$$p = \sqrt{\frac{E^2 - (E - KE)^2}{c^2}} = \sqrt{\frac{(5.0 \times 10^{15} \text{ J})^2 - [5.0 \times 10^{15} \text{ J} - (2.0 \times 10^{15} \text{ J})]^2}{(3.0 \times 10^8 \text{ m/s})^2}} = 1.3 \times 10^7 \text{ kg \cdot m/s}$$

34. **REASONING** If the speeds of your car and the truck are much less than the speed of light, the relative speed at which the truck approaches you is the same in parts (a) and (b). Let’s suppose that you are traveling due east, which is taken to be the positive direction, and the truck is traveling due west, which is the negative direction. The relative velocities are:

$$v_{TC} = \text{velocity of the Truck relative to the Car}$$
$$v_{TG} = \text{velocity of the Truck relative to the Ground}$$
$$v_{CG} = \text{velocity of the Car relative to the Ground}$$

(Note that the velocity $v_{GC}$ of the Ground relative to the Car is $v_{GC} = -v_{CG}$.)

The velocity $v_{TC}$ of the truck with respect to the car is equal to the velocity $v_{TG}$ of the truck with respect to the ground plus the velocity $v_{GC}$ of the ground with respect to the car: $v_{TC} = v_{TG} + v_{GC}$, as discussed in Section 3.4.

When $v_{TG} = -35 \text{ m/s}$ and $v_{GC} = -25 \text{ m/s}$, the velocity of the truck relative to the car is $v_{TC} = -60 \text{ m/s}$, where the minus sign indicates that the relative velocity is westward.
SPECIAL RELATIVITY

When \( v_{TG} = -55 \text{ m/s} \) and \( v_{CG} = +5.0 \text{ m/s} \), the relative velocity of the truck with respect to the car is still \( v_{TC} = v_{TG} + v_{GC} = -55 \text{ m/s} - 5 \text{ m/s} = -60 \text{ m/s} \). In either case, the speed is the magnitude of \( v_{TC} \), or 60 m/s.

However, the relative velocities and, hence, the relative speeds would not be the same in parts (a) and (b) if the speeds were comparable to the speed of light. According to special relativity, the correct relation is the velocity-addition formula, Equation 28.8:

\[
v_{TC} = \frac{v_{TG} + v_{GC}}{1 + \frac{v_{TG}v_{GC}}{c^2}}
\]

Because of the presence of the term \( v_{TG}v_{GC}/c^2 \) in the denominator, different results are obtained when \( v_{TG} = -35 \text{ m/s} \) and \( v_{GC} = -25 \text{ m/s} \) than when \( v_{TG} = -55 \text{ m/s} \) and \( v_{GC} = -5.0 \text{ m/s} \).

**SOLUTION**

a. When \( v_{TG} = -35 \text{ m/s} \) and \( v_{GC} = -25 \text{ m/s} \), the velocity of the truck relative to the car is

\[
v_{TC} = \frac{-35 \text{ m/s} - 25 \text{ m/s}}{1 + \frac{(-35 \text{ m/s})(-25 \text{ m/s})}{(65 \text{ m/s})^2}} = -49.7 \text{ m/s}
\]

The speed of the truck relative to the car is the magnitude of this result, or \( 49.7 \text{ m/s} \).

b. When \( v_{TG} = -55 \text{ m/s} \) and \( v_{GC} = -5.0 \text{ m/s} \), the velocity of the truck relative to the car is

\[
v_{TC} = \frac{-55 \text{ m/s} - 5.0 \text{ m/s}}{1 + \frac{(-55 \text{ m/s})(-5.0 \text{ m/s})}{(65 \text{ m/s})^2}} = -56.3 \text{ m/s}
\]

The speed of the truck relative to the car is the magnitude of this result, or \( 56.3 \text{ m/s} \).

---

**SSM REASONING** Let’s define the following relative velocities, assuming that the spaceship and exploration vehicle are moving in the positive direction.

\[ v_{ES} = \text{velocity of Exploration vehicle relative to the Spaceship.} \]
\[ v_{EO} = \text{velocity of Exploration vehicle relative to an Observer on earth} = +0.70c \]
\[ v_{SO} = \text{velocity of Spaceship relative to an Observer on earth} = +0.50c \]

The velocity \( v_{ES} \) can be determined from the velocity-addition formula, Equation 28.8:
The velocity $v_{OS}$ of the observer on earth relative to the spaceship is not given. However, we know that $v_{OS}$ is the negative of $v_{SO}$, so $v_{OS} = -v_{SO} = -(+0.50c) = -0.50c$.

**SOLUTION** The velocity of the exploration vehicle relative to the spaceship is

$$v_{ES} = \frac{v_{EO} + v_{OS}}{1 + \frac{v_{EO}v_{OS}}{c^2}}$$

The speed of the exploration vehicle relative to the spaceship is the magnitude of this result or $0.31c$.

36. **REASONING** We assume that the direction away from the earth is the positive direction. With this assumption, we have the following relative velocities:

- $v_{IE}$ = the velocity of Ions relative to Earth
- $v_{IS}$ = the velocity of Ions relative to the Spaceship = $-0.80c$.
- $v_{SE}$ = the velocity of Spaceship relative to Earth = $+0.70c$

Note that the velocity $v_{IE}$ of the ions relative to the spaceship is negative because the spaceship is moving away from the earth (in the positive direction), and the ions are emitted from the engine in the opposite or negative direction. It is the velocity $v_{IE}$ that we seek.

**SOLUTION** These velocities defined in the **REASONING** are related by the velocity-addition formula, Equation 28.7, according to which we have

$$v_{IE} = \frac{v_{IS} + v_{SE}}{1 + \frac{v_{IS}v_{SE}}{c^2}} = \frac{-0.80c + 0.70c}{1 + \frac{(-0.80c)(+0.70c)}{c^2}} = -0.23c$$

37. **REASONING** The velocity of the Enterprise 2, as measured by an earth-based observer, is given by

$$v_{2e} = \frac{v_{21} + v_{1e}}{1 + \frac{v_{21}v_{1e}}{c^2}}$$

where

- $v_{2e}$ = velocity of Enterprise 2 relative to earth
- $v_{21}$ = velocity of Enterprise 2 relative to Enterprise 1
- $v_{1e}$ = velocity of Enterprise 1 relative to earth


All of these variables are known, so $v_{2e}$ can be determined.

**SOLUTION** The velocity of *Enterprise* 2 relative to the earth is

$$v_{2e} = \frac{v_{21} + v_{1e}}{1 + \frac{v_{21} v_{1e}}{c^2}} = \frac{(+0.31c) + (+0.65c)}{1 + \frac{(0.31c)(0.65c)}{c^2}} = +0.80c$$

38. **REASONING** We define the following relative velocities, assuming that the rocket approaching the earth from the right is traveling in the positive direction:

$v_{RL} =$ velocity of the *Right* rocket relative to the *Left* rocket

$v_{RE} =$ velocity of the *Right* rocket relative to the person on *Earth* = +0.75$c$

$v_{LE} =$ velocity of the *Left* rocket relative to the person on *Earth* = –0.65$c$

The velocity $v_{RL}$ can be found from the velocity-addition formula, Equation 28.8:

$$v_{RL} = \frac{v_{RE} + v_{EL}}{1 + \frac{v_{RE} v_{EL}}{c^2}}$$

The velocity $v_{RE}$ is given, but $v_{EL}$, the velocity of the earth relative to the left rocket, is not. However, we know that $v_{EL}$ is the negative of $v_{LE}$, so $v_{EL} = -v_{LE} = -(–0.65c) = +0.65c$.

**SOLUTION** The velocity of the right rocket relative to the left rocket is

$$v_{RL} = \frac{v_{RE} + v_{EL}}{1 + \frac{v_{RE} v_{EL}}{c^2}} = \frac{+0.75c + 0.65c}{1 + \frac{(0.75c)(0.65c)}{c^2}} = +0.94c$$

The relative speed between the two rockets is the magnitude of this result, or $0.94c$.

39. **SSM** **REASONING AND SOLUTION**

   In all parts of this problem, the direction of the intergalactic cruiser, the ions, and the laser light is taken to be the positive direction.

   a. According to the second postulate of special relativity, all observers measure the speed of light to be $c$, regardless of their velocities relative to each other. Therefore, the aliens aboard the hostile spacecraft see the photons of the laser approach **at the speed of light,** $c$.

   b. To find the velocity of the ions relative to the aliens, we define the relative velocities as follows:
Chapter 28  Problems  1425

\( v_{IS} \) = velocity of the ions relative to the alien spacecraft

\( v_{IC} \) = velocity of the ions relative to the intergalactic cruiser = +0.950c

\( v_{CS} \) = velocity of the intergalactic cruiser relative to the alien spacecraft = +0.800c

These velocities are related by the velocity-addition formula, Equation 28.8. The velocity of the ions relative to the alien spacecraft is:

\[
v_{IS} = \frac{v_{IC} + v_{CS}}{1 + \frac{v_{IC}v_{CS}}{c^2}} = \frac{+0.950c + 0.800c}{1 + \frac{(+0.950c)(+0.800c)}{c^2}} = +0.994c
\]

c. The aliens see the laser light (photons) moving with respect to the cruiser at a velocity

\[ U = +1.000c - 0.800c = +0.200c \]

d. The aliens see the ions moving away from the cruiser at a velocity

\[ U' = +0.994c - 0.800c = +0.194c \]

40. REASONING  The passengers would measure a length for the spaceships that does not match the constructed length if the two ships have a nonzero relative speed. This would mean that the ships are moving with respect to one another. If so, the phenomenon of length contraction would occur, and the passengers in either ship would measure a contracted length.

To calculate the contracted length \( L \), the length contraction formula must be used, as given in Equation 28.2:

\[
L = L_0\sqrt{1 - \frac{v_{AB}^2}{c^2}} \quad (28.2)
\]

where \( L_0 \) is the proper length of the spaceships, that is, the length measured by an observer at rest with respect to them. In other words, the proper length is the constructed length. The velocity \( v_{AB} \) in Equation 28.2 is the velocity of spaceship A with respect to spaceship B. It can be obtained from the velocities of each ship with respect to the earth by using the velocity addition equation, as given in Equation 28.8:

\[
v_{AB} = \frac{v_{AE} + v_{EB}}{1 + \frac{v_{AE}v_{EB}}{c^2}} \quad (28.8)
\]

where \( v_{AE} \) is the velocity of spaceship A with respect to the earth and \( v_{EB} \) is the velocity of the earth with respect to spaceship B. We note that \( v_{EB} = -v_{BE} \), where \( v_{BE} \) is the velocity of spaceship B with respect to the earth. Substituting \( v_{EB} = -v_{BE} \) into Equation 28.8 gives
\[ v_{AB} = \frac{v_{AE} - v_{BE}}{1 - \frac{v_{AE}v_{BE}}{c^2}} \quad (1) \]

**SOLUTION** Using Equation (1) to calculate \( v_{AB} \) for use in Equation 28.2, we find

\[
v_{AB} = \frac{0.850c - 0.500c}{1 - \frac{(0.850c)(0.500c)}{c^2}} = 0.609c
\]

Here we have taken the direction in which the spaceships are traveling to be the positive direction. Substituting this result into Equation 28.2 reveals that

\[
L = L_0 \sqrt{1 - \frac{v_{AB}^2}{c^2}} = (1.50 \text{ km}) \sqrt{1 - \frac{(0.609c)^2}{c^2}} = 1.19 \text{ km}
\]

41. **REASONING**

The following relative velocities are pertinent to this problem. It is assumed that particle 1 is moving in the positive direction, and, therefore, particle 2 is moving in the negative direction.

- \( v_{P_1P_2} = \) velocity of particle 1 \((P_1)\) relative to particle 2 \((P_2)\)
- \( v_{P_1L} = \) velocity of particle 1 \((P_1)\) relative to an observer in the Laboratory
  - \( = +2.10 \times 10^8 \text{ m/s} \)
- \( v_{P_2L} = \) velocity of particle 2 \((P_2)\) relative to an observer in the Laboratory
  - \( = -2.10 \times 10^8 \text{ m/s} \)

The velocity \( v_{P_1P_2} \) can be obtained from the velocity-addition formula, Equation 28.8:

\[
v_{P_1P_2} = \frac{v_{P_1L} + v_{LP_2}}{1 + \frac{v_{P_1L}v_{LP_2}}{c^2}}
\]

The velocity \( v_{P_1L} \) is given, but \( v_{LP_2} \), the velocity of the laboratory observer relative to particle 2, is not. However, we know that \( v_{LP_2} \) is the negative of \( v_{P_2L} \), so \( v_{LP_2} = -v_{P_2L} = -(2.10 \times 10^8 \text{ m/s}) = +2.10 \times 10^8 \text{ m/s} \).

**SOLUTION**

a. According to the velocity-addition formula, the velocity of particle 1 relative to particle 2 is

\[
v_{P_1P_2} = \frac{v_{P_1L} + v_{LP_2}}{1 + \frac{v_{P_1L}v_{LP_2}}{c^2}} = \frac{+2.10 \times 10^8 \text{ m/s} + 2.10 \times 10^8 \text{ m/s}}{1 + \frac{(2.10 \times 10^8 \text{ m/s})(2.10 \times 10^8 \text{ m/s})}{c^2}} = +2.82 \times 10^8 \text{ m/s}
\]
The speed of one particle as seen by the other particle is the magnitude of this result, or \(2.82 \times 10^8 \text{ m/s}\).

b. The relativistic momentum is given by Equation 28.3, where the speed is that determined in part a. Therefore,

\[
p = \frac{mv_p p_z}{\sqrt{1 - \left(\frac{v_p p_z}{c}\right)^2}} = \frac{\left(2.16 \times 10^{-25} \text{ kg}\right)\left(+2.82 \times 10^8 \text{ m/s}\right)}{\sqrt{1 - \left(\frac{2.82 \times 10^8 \text{ m/s}}{3.00 \times 10^8 \text{ m/s}}\right)^2}} = 1.8 \times 10^{-16} \text{ kg} \cdot \text{m/s}
\]

42. **REASONING** The total energy \(E\) for each particle is \(E = \frac{mc^2}{\sqrt{1 - v^2 / c^2}}\) (Equation 28.4), where \(m = 9.11 \times 10^{-31} \text{ kg}\) is the mass of each particle, \(v = 0.20c\) is the speed of each particle, and \(c\) is the speed of light in a vacuum. The total energy of the electromagnetic radiation that appears after the collision is twice the value given by Equation 28.4. Note that we will not use \(E_0 = mc^2\) (Equation 28.5), which gives the rest energy \(E_0\), because the particles are moving and are not at rest.

**SOLUTION** Applying Equation 28.4 for the total energy of each particle, we find that the energy of the electromagnetic radiation that appears after the collision is

\[
E_{\text{electromagnetic radiation}} = \frac{mc^2}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} + \frac{mc^2}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \frac{2mc^2}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \frac{2\left(9.11 \times 10^{-31} \text{ kg}\right)\left(3.00 \times 10^8 \text{ m/s}\right)^2}{\sqrt{1 - \left(\frac{0.20c}{c}\right)^2}} = 1.7 \times 10^{-13} \text{ J}
\]

43. **SSM REASONING** Assume that traveler A moves at a speed of \(v_A = 0.70c\) and traveler B moves at a speed of \(v_B = 0.90c\), both speeds being with respect to the earth. Each traveler is moving with respect to the earth and the distant star, so each measures a contracted length \(L_A\) or \(L_B\) for the distance traveled. However, an observer on earth is at rest with respect to the earth and the distant star (which is assumed to be stationary with respect to the earth), so he or she would measure the proper length \(L_0\). For each traveler the contracted length is given by the length-contraction equation as stated in Equation 28.2:
\[ L_A = L_0 \sqrt{1 - \frac{v_A^2}{c^2}} \quad \text{and} \quad L_B = L_0 \sqrt{1 - \frac{v_B^2}{c^2}} \]

It is important to note that the proper length \( L_0 \) is the same in each application of the length-contraction equation. Thus, we can combine the two equations and eliminate it. Then, since we are given values for \( L_A, v_A, \) and \( v_B, \) we will be able to determine \( L_B. \)

**SOLUTION** Dividing the expression for \( L_B \) by the expression for \( L_A \) and eliminating \( L_0, \)

\[ \frac{L_B}{L_A} = \frac{L_0 \sqrt{1 - \frac{v_B^2}{c^2}}}{L_0 \sqrt{1 - \frac{v_A^2}{c^2}}} = \frac{\sqrt{1 - \frac{v_B^2}{c^2}}}{\sqrt{1 - \frac{v_A^2}{c^2}}} \]

\[ L_B = L_A \sqrt{\frac{1 - \frac{v_B^2}{c^2}}{1 - \frac{v_A^2}{c^2}}} = (6.5 \text{ light-years}) \sqrt{\frac{1 - (0.90c)^2}{1 - (0.70c)^2}} = 4.0 \text{ light-years} \]

44. **REASONING** The relativistic momentum \( p \) of an object of mass \( m \) is given by

\[ p = mv / \sqrt{1 - (v^2 / c^2)} \] (Equation 28.3), where \( v \) is the speed of the object and \( c \) is the speed of light in a vacuum. For part (a), we will solve Equation 28.3 to determine the mass of the ion. In part (b), we will use the mass determined in part (a) in Equation 28.3 to calculate the relativistic momentum of the ion at its final speed.

**SOLUTION**

a. Solving Equation 28.3 for \( m \) and using the initial speed of \( v = 0.460c, \) we obtain

\[ m = \frac{p \sqrt{1 - \frac{v^2}{c^2}}}{v} = \frac{(5.08 \times 10^{-17} \text{ kg} \cdot \text{m/s}) \sqrt{1 - \left(\frac{0.460c}{c}\right)^2}}{0.460 \left(3.00 \times 10^8 \text{ m/s}\right)} = 3.27 \times 10^{-25} \text{ kg} \]

b. Substituting the value for \( m \) found in part (a) and the final speed of \( v = 0.920c \) into Equation 28.3 yields
\[ p = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{(3.27 \times 10^{-25} \text{ kg})(0.920)(3.00 \times 10^8 \text{ m/s})}{\sqrt{1 - \left(\frac{0.920c}{c}\right)^2}} = 2.30 \times 10^{-16} \text{ kg} \cdot \text{m/s} \]

45. **REASONING** The expression for time dilation is, according to Equation 28.1,

\[ \Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \]

For a given event, it relates the proper time interval \( \Delta t_0 \) to the time interval \( \Delta t \) that would be measured by an observer moving at a speed \( v \) relative to the frame of reference in which the event takes place.

We must consider two situations; in the first situation, the Klingon spacecraft has a speed of 0.75\( c \) with respect to the earth. In the second situation, the craft has a speed of 0.94\( c \) relative to the earth. We will refer to these two situations as A and B, respectively.

Since the proper time interval always has the same value, \( (\Delta t_0)_A = (\Delta t_0)_B \). We can express both sides of this expression using Equation 28.1. The result can be solved for \( \Delta t_B \).

**SOLUTION** Use of Equation 28.1 gives

\[ \Delta t_A \sqrt{1 - \frac{v_A^2}{c^2}} = \Delta t_B \sqrt{1 - \frac{v_B^2}{c^2}} \]

\[ \Delta t_B = \Delta t_A \frac{\sqrt{1 - \frac{v_A^2}{c^2}}}{\sqrt{1 - \frac{v_B^2}{c^2}}} = \Delta t_A \frac{1 - (v_A / c)^2}{1 - (v_B / c)^2} = (37.0 \text{ h}) \frac{1 - (0.75c / c)^2}{1 - (0.94c / c)^2} = 72 \text{ h} \]

46. **REASONING** The total linear momentum of the system is conserved, since no net external force acts on the system. Therefore, the final total momentum \( p_1 + p_2 \) of the two fragments must equal the initial total momentum, which is zero since the particle is initially at rest. As a result, \( p_1 = -p_2 \), where Equation 28.3 must be used for the magnitudes of the momenta \( p_1 \) and \( p_2 \). Thus, we find

\[ \frac{m_1 v_1}{\sqrt{1 - (v_1^2 / c^2)}} = \frac{-m_2 v_2}{\sqrt{1 - (v_2^2 / c^2)}} \]

**SOLUTION** Letting fragment 2 be the more-massive fragment, we have that \( v_1 = +0.800c \), \( m_1 = 1.67 \times 10^{-27} \text{ kg} \), and \( m_2 = 5.01 \times 10^{-27} \text{ kg} \). Squaring both sides of the above equation, rearranging terms, substituting the known values for \( v_1, m_1, \) and \( m_2 \), we find that
\[ \frac{v_2^2}{1-(v_2^2/c^2)} = \frac{m_1^2 v_1^2}{m_2^2 \left[1-(v_1^2/c^2)\right]} = \frac{\left(1.67 \times 10^{-27} \text{ kg}\right)^2 \left(+0.800c\right)^2}{\left(5.01 \times 10^{-27} \text{ kg}\right)^2 \left[1-\left(+0.800c/c\right)^2\right]} = 0.1975c^2 \]

Therefore,

\[ v_2^2 = 0.1975c^2 \left[1-(v_2^2/c^2)\right] = 0.1975c^2 - 0.1975v_2^2 \]

Solving for \( v_2 \) gives

\[ v_2 = \pm \sqrt{\frac{0.1975c^2}{1.1975}} = \pm 0.406c \]

We reject the positive root, since then both fragments would be moving in the same direction after the break-up and the system would have a non-zero momentum. According to the principle of conservation of momentum, the total momentum after the break-up must be zero, just as it was before the break-up. The momentum of the system will be zero only if the velocity \( v_2 \) is opposite to the velocity \( v_1 \). Hence, we chose the negative root and

\[ v_2 = -0.406c \]

47. **SSM REASONING** Since the crew is initially at rest relative to the escape pod, the length of 45 m is the proper length \( L_0 \) of the pod. The length of the escape pod as determined by an observer on earth can be obtained from the relation for length contraction given by Equation 28.2, \( L = L_0\sqrt{1-\left(v_{PE}/c\right)^2} \). The quantity \( v_{PE} \) is the speed of the escape pod relative to the earth, which can be found from the velocity-addition formula, Equation 28.8. The following are the relative velocities, assuming that the direction away from the earth is the positive direction:

\[ v_{PE} = \text{velocity of the escape Pod relative to Earth.} \]
\[ v_{PR} = \text{velocity of escape Pod relative to the Rocket} = -0.55c. \] This velocity is negative because the rocket is moving away from the earth (in the positive direction), and the escape pod is moving in an opposite direction (the negative direction) relative to the rocket.
\[ v_{RE} = \text{velocity of Rocket relative to Earth} = +0.75c \]

These velocities are related by the velocity-addition formula, Equation 28.8.

**SOLUTION** The relative velocity of the escape pod relative to the earth is

\[ v_{PE} = \frac{v_{PR} + v_{RE}}{1 + \frac{v_{PR} v_{RE}}{c^2}} = \frac{-0.55c + 0.75c}{1 + \frac{(-0.55c)(+0.75c)}{c^2}} = +0.34c \]
The speed of the pod relative to the earth is the magnitude of this result, or 0.34c. The length of the pod as determined by an observer on earth is

$$L = L_0 \sqrt{1 - \frac{v^{2}}{c^{2}}} = (45 \text{ m}) \sqrt{1 - \frac{(0.34c)^{2}}{c^{2}}} = 42 \text{ m}$$

48. **REASONING** The total relativistic energy $E$ is related to the rest energy $E_0 = mc^2$ and the speed $v$ according to Equation 28.4:

$$E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{E_0}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (28.4)$$

A greater total energy $E$ does not necessarily mean that an object has a greater speed. It is not the total energy alone that matters, but the ratio $E/E_0$ of the total energy to the rest energy. According to Equation 28.4 this ratio is

$$\frac{E}{E_0} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (1)$$

Larger values for this ratio mean that the speed $v$ is greater. When the speed is greater, square root term in the denominator on the right-hand side of Equation (1) is smaller, so the reciprocal of the square root term is larger.

By considering the ratio given in Equation (1), we can rank the speeds of the objects. This ratio is listed in the following table:

<table>
<thead>
<tr>
<th>Object</th>
<th>Total Energy ($E$)</th>
<th>Rest Energy ($E_0$)</th>
<th>$E/E_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2.00ε</td>
<td>ε</td>
<td>2.00</td>
</tr>
<tr>
<td>B</td>
<td>3.00ε</td>
<td>ε</td>
<td>3.00</td>
</tr>
<tr>
<td>C</td>
<td>3.00ε</td>
<td>2.00ε</td>
<td>1.50</td>
</tr>
</tbody>
</table>

The ratio $E/E_0$ is greatest for object B and smallest for object C. Thus, the ranking of the speeds is

B (largest), A, C

**SOLUTION** Solving Equation (1) for the speed $v$, we obtain
\[ \frac{E}{E_0} = \sqrt{1 - \frac{v^2}{c^2}} \] or \( \left( \frac{E}{E_0} \right)^2 = 1 - \frac{v^2}{c^2} \) or \[ \frac{1}{(E/E_0)^2} = 1 - \frac{v^2}{c^2} \] or \[ v = c \sqrt{1 - \frac{1}{(E/E_0)^2}} \]

Applying this result to each object, we find

\[ v_A = c \sqrt{1 - \frac{1}{(E/E_0)^2}_A} = c \sqrt{1 - \frac{1}{2.00^2}} = 0.866c \]

\[ v_B = c \sqrt{1 - \frac{1}{(E/E_0)^2}_B} = c \sqrt{1 - \frac{1}{3.00^2}} = 0.943c \]

\[ v_C = c \sqrt{1 - \frac{1}{(E/E_0)^2}_C} = c \sqrt{1 - \frac{1}{1.50^2}} = 0.745c \]

These results are consistent with our expected ranking. In particular, note that the largest total energy for object C does not imply that its speed is the largest.

49. **REASONING** The first twin, traveling at the higher speed \( v_1 = 0.900c \), arrives at the distant planet first. Thereafter, the first twin is at rest relative to the earth, and ages at the same rate as people back on the earth. As measured by an observer on the earth, the dilated time intervals \( \Delta t_1, \Delta t_2 \) for the journeys of the two twins are related to the proper time intervals \( \Delta t_{01}, \Delta t_{02} \) for their journeys by

\[ \Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \] (Equation 28.1), so we have that

\[ \Delta t_{01} = \Delta t_1 \sqrt{1 - \frac{v_1^2}{c^2}} \] and \[ \Delta t_{02} = \Delta t_2 \sqrt{1 - \frac{v_2^2}{c^2}} \]

(1)

The proper time intervals \( \Delta t_{01} \) and \( \Delta t_{02} \) in Equations (1) give the aging experienced by each twin during the journey. The additional aging that the first twin undergoes between the time of his arrival at the distant planet and the arrival of the second twin is the difference between the time of the second twin’s journey and the time of the first twin’s journey, both as measured by an observer on the earth:

\[ \text{Additional aging} = \Delta t_2 - \Delta t_1 \]

(2)

Lastly, we will use \( v = \frac{d}{\Delta t} \) (Equation 2.1) to determine the time intervals for each twin’s journey as measured from the earth, where \( d = 12.0 \) light-years is the distance between the earth and the distant planet. The distance \( d \) and the time intervals \( \Delta t_1, \Delta t_2 \) are all measured
by an observer on the earth, so we do not need to apply the special theory of relativity. Therefore, we have that
\[ \Delta t_1 = \frac{d}{v_1} \quad \text{and} \quad \Delta t_2 = \frac{d}{v_2} \]

**SOLUTION**

a. When the second twin arrives at the distant planet, the final age of the first twin is given by \( A_1 = A + \Delta t_{01} + \Delta t_2 - \Delta t_1 \), where \( A = 19.0 \) years is the initial age of both twins and we have used Equation (2) to take into account the additional aging. The second twin’s final age is the initial age \( A \) of the twin plus the time \( \Delta t_{02} \) that elapses during the journey, as measured by that twin: \( A_2 = A + \Delta t_{02} \). Therefore, the net difference between their ages will be

\[ A_2 - A_1 = A + \Delta t_{02} - (A + \Delta t_{01} + \Delta t_2 - \Delta t_1) = \Delta t_{02} - \Delta t_{01} - \Delta t_2 + \Delta t_1 \]

Substituting Equations (1) into Equation (4) yields

\[ A_2 - A_1 = \Delta t_2 \left( \sqrt{1 - \frac{v_2^2}{c^2}} - 1 \right) - \Delta t_1 \left( \sqrt{1 - \frac{v_1^2}{c^2}} - 1 \right) \]

Substituting Equations (3) into Equation (5), we obtain

\[ A_2 - A_1 = \Delta t_2 \left( \sqrt{1 - \frac{v_2^2}{c^2}} - 1 \right) - \Delta t_1 \left( \sqrt{1 - \frac{v_1^2}{c^2}} - 1 \right) = \frac{d}{v_2} \left( \sqrt{1 - \frac{v_2^2}{c^2}} - 1 \right) - \frac{d}{v_1} \left( \sqrt{1 - \frac{v_1^2}{c^2}} - 1 \right) \]

In applying Equation (6), we make use of the fact that the speed \( c \) of light in a vacuum is equal to 1 light-year per year: \( c = 1 \) light-year/year:

\[ A_2 - A_1 = \frac{12.0 \text{ light-years}}{0.500 \text{ light-years/year}} \left( \sqrt{1 - \frac{(0.500 c)^2}{c^2}} - 1 \right) \]

\[ - \frac{12.0 \text{ light-years}}{0.900 \text{ light-years/year}} \left( \sqrt{1 - \frac{(0.900 c)^2}{c^2}} - 1 \right) = 4.3 \text{ years} \]

b. Since \( A_2 - A_1 \) is positive, \( A_2 \) is greater than \( A_1 \). Thus, when the twins meet again at the earliest possible time, the second twin (traveling at 0.500 \( c \)) is older.