Solve each triangle. For each problem, draw a triangle and label all three angles and all three sides. If there are two solutions, solve and draw both triangles. If no triangles exist, write "No Triangles Exist." Round all angles to the nearest degree and all sides to the nearest tenth.

1)   Given $\angle A = 47°$, $\angle B = 63°$, and c = 123.

2)   Given a = 123, c = 86, and $\angle C = 52°$.

3)   Given $\angle B = 78°$, c = 142, and b = 120.

4)   Given a = 18, b = 23, and $\angle B = 97°$.

5)   Given a = 56, b = 71, and $\angle B = 100°$.

6)   Given $\angle A = 56°$, a = 112, and b = 130.

7)   Given $\angle B = 61°$, c = 18, and b = 17

8)   a = 17, b = 39, and c = 50.

9)   a = 78, b = 15, and c = 91.

10) $\angle A = 58°20'$, b = 23, and c = 18.

Draw a diagram and use the law of sines or the law of cosines to solve each problem.

11)   A television antenna sits on the roof. Two 78 ft guy wires are positioned on opposite sides of the antenna. The angle of elevation each makes with the ground is 23°. How far apart are the ends of the two guy wires? (143.6 feet)

12)   From fire tower A a fire with bearing N 78° E is sighted. The same fire is sighted from tower B at N 51° W. Tower A is 70 miles west of tower B. How far is it from tower A to the fire? (56.7 miles)

13)   Ship A is 72 miles from a lighthouse on the shore. Its bearing from the lighthouse is N 15° E. Ship B is 81 miles from the same lighthouse. Its bearing from the lighthouse is N 52° E. Find the number of miles between the two ships. (49.29)

14)   In order to determine the distance between two aircraft, a tracking station continuously monitors the distance to each aircraft and the angle α between them. Determine the distance between the planes when $\alpha = 28°$, distance to one plane is 71 miles and the distance to the other plane is 36 miles. (42.7 miles)

Area of an Oblique Triangle

1. Find the area of a triangle with sides of length 7 and 9 and included angle 72°. (30)

2. Find the area of an equilateral triangle with side of length 10. (43.3)

3. A Heron Triangle is a triangle having integer sides and area. Show that each of the following are Heron Triangles:
   (a) a = 11, b = 13, c = 20
   (b) a = 13, b = 14, c = 15
   (c) a = 7, b = 15, c = 20

4. A Perfect Triangle is a triangle whose sides have whole number lengths and whose area is numerically equivalent to its perimeter. Show that a triangle with side lengths 9, 10, and 17 is perfect.

5. To the nearest square foot, what is the area of a triangular pond, if two adjacent sides measure 112 ft and 96 ft and the angle between them measures 65.4°? (4888 sq ft)

6. What is the area, to the nearest 0.1 acre, of a triangular field that is 529 ft on one side and 723 ft on another, if the angle between these sides measures 102.7°? 1 acre = 43,560 sq ft. (4.3 acres)

7. Land in downtown Columbia is valued at $20 a square foot. What is the value of a triangular lot with sides of lengths 112, 148, and 190 ft? ($165,554)
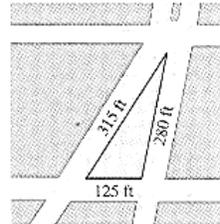
8. Find the area of the quadrilateral in the figure. (40.77)
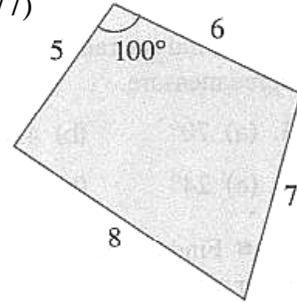
9. Find the area of the shaded region in the figure. (2.47)
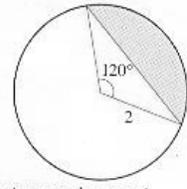
10. A businessman wishes to buy a triangular lot in a busy downtown location. The lot frontages on the three streets are 125, 280, and 315 ft. Find the area of the lot. (17,452 square feet)

11. Three circles of radii 3, 4, and 5 are mutually tangent. Find the area of the enclosed region between the circles. (2.39)